Problem 1.56

The Hydrogen Maser. You can use the radio waves generated by a hydrogen maser as a standard of frequency. The frequency of these waves is 1,420,405,751.786 hertz. (A hertz is another name for one cycle per second.) A clock controlled by a hydrogen maser is off by only 1 s in 100,000 years. For the following questions, use only three significant figures. (The large number of significant figures given for the frequency simply illustrates the remarkable accuracy to which it has been measured.) (a) What is the time for one cycle of the radio wave? (b) How many cycles occur in 1 h? (c) How many cycles would have occurred during the age of the earth, which is estimated to be 4.6×10^9 years? (d) By how many seconds would a hydrogen maser clock be off after a time interval equal to the age of the earth?

Solution

The given frequency of the radio waves is

$$\nu = 1\,420\,405\,751.786\ \frac{\text{cycles}}{\text{second}}.$$

Part (a)

Invert it to get the number of seconds in one cycle.

$$\frac{1}{\nu} = \frac{1}{1\,420\,405\,751.786}\ \frac{\text{seconds}}{\text{cycle}} \approx 7.04 \times 10^{-10}\ \frac{\text{seconds}}{\text{cycle}}$$

Part (b)

Use known conversion factors to get the number of cycles in one hour.

$$\nu = 1\,420\,405\,751.786\ \frac{\text{cycles}}{\cancel{\text{second}}} \times \frac{60\ \cancel{\text{sec}}}{1\ \cancel{\text{min}}} \times \frac{60\ \cancel{\text{min}}}{1\ \text{hour}} \approx 5.11 \times 10^{12}\ \frac{\text{cycles}}{\text{hour}}$$

Part (c)

Convert the age of the earth to hours

$$4.6 \times 10^9\ \cancel{\text{years}} \times \frac{365\ \cancel{\text{days}}}{1\ \cancel{\text{year}}} \times \frac{24\ \text{hours}}{1\ \cancel{\text{day}}} \approx 4.03 \times 10^{13}\ \text{hours}$$

and then multiply it by the result of part (b) to get the number of cycles.

$$5.11 \times 10^{12}\ \frac{\text{cycles}}{\cancel{\text{hour}}} \times 4.03 \times 10^{13}\ \cancel{\text{hours}} \approx 2.06 \times 10^{26}\ \text{cycles}$$

Part (d)

$$4.6 \times 10^9\ \cancel{\text{years}} \times \frac{1\ \text{s}}{100\,000\ \cancel{\text{years}}} = 4.60 \times 10^4\ \text{seconds}$$